Name: _____ **Teacher: MN MA GS LR RC SW**

NESA Number: _____

ASCHAM SCHOOL

MATHEMATICS ADVANCED TRIAL EXAMINATION 2022

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black non-erasable pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper

Section I **10 marks**

- Answer Questions 1–10 using the Multiple Choice sheet
- Allow about 15 minutes for this section.

Section II **90 marks**

- Attempt Questions 11–35.
- Allow about 2 hours 45 minutes for this section.
- For questions in Section II, show relevant mathematical reasoning and/ or calculations.
- Additional writing space is available on pages 39 to 43. If you use this space, please start a new sheet of paper for each question.

Section I

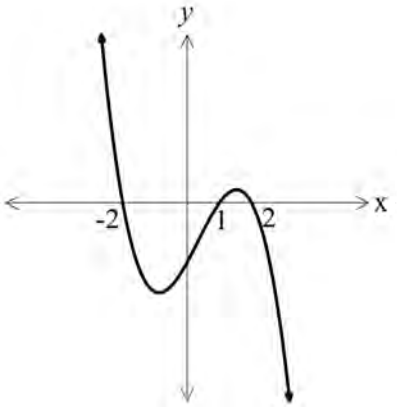
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

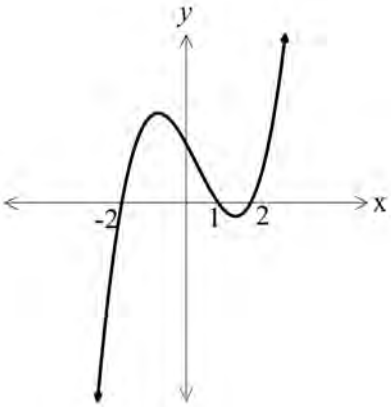
---

1 Which of the following could be the graph of $y = (x + 2)(x - 2)(1 - x)$?

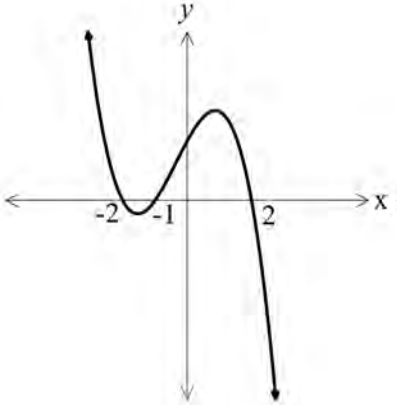
(A)



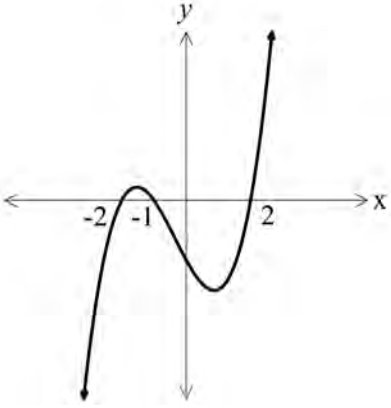
(B)



(C)



(D)



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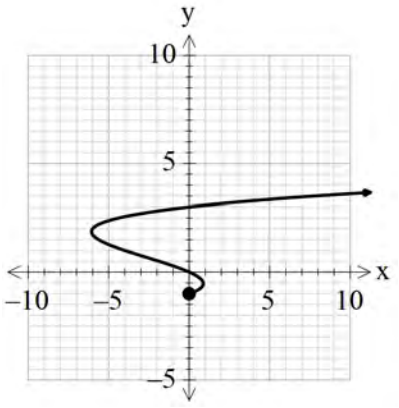

(A) One-to-one

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(D) Many-to-many

3. One card is selected at random from a pack of cards labelled 1 to 50. Find the probability that the card is an odd number or a number greater than 40.

(A) 0.1 (B) 0.58 (C) 0.6 (D) 0.7

4 The function $y = f(x)$ is transformed to $y = f\left(\frac{1}{2}(x+2)\right)$.

Which of the following describes the transformations that took place?

(A) Horizontal dilation factor of 2, followed by translation left 1 unit.

(B) Horizontal dilation factor of 2, followed by translation left 2 units.

(C) Horizontal dilation factor of $\frac{1}{2}$, followed by translation left 1 unit.

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5. Which of the following is true for the function $y = 2\cos\left(\frac{x}{3}\right) + 4$

(A) Period $= 6\pi$, Range: $[-6, 6]$

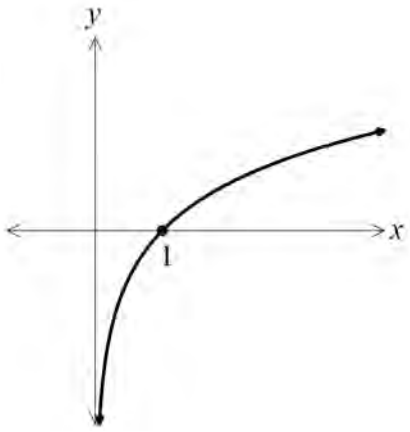
(B) Period $= 6\pi$, Range: $[2, 6]$

(C) Period $= \frac{2\pi}{3}$, Range: $[-6, 6]$

(D) Period $= \frac{2\pi}{3}$, Range: $[2, 6]$

6. Which of the following could be the graph of the function $y = 2\log_e\left(\frac{1}{x}\right) + 1$

(A)



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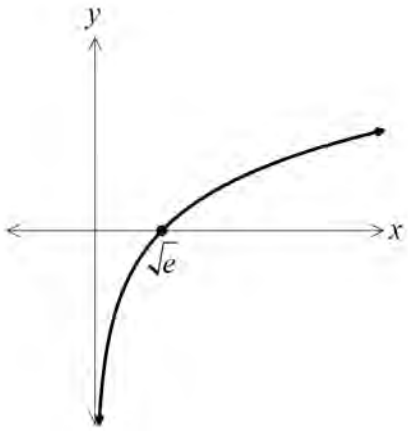

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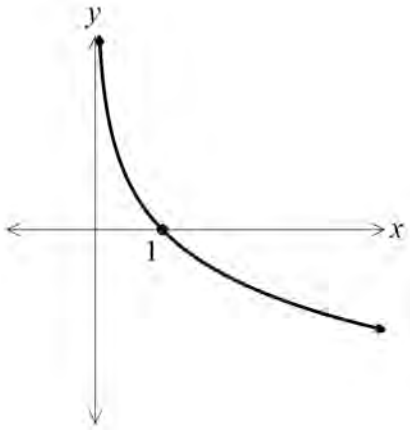
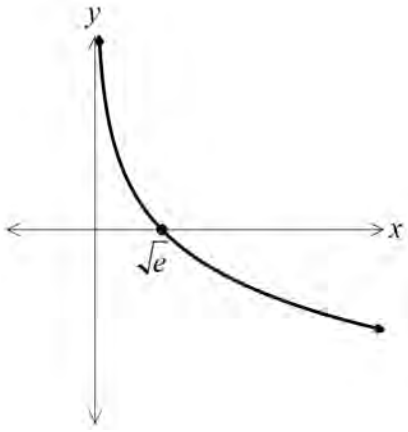
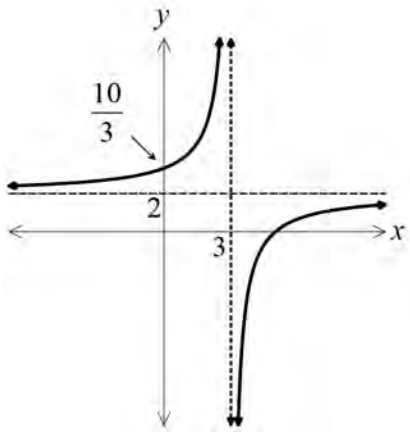


(D)



7. Consider the graph of $y = \dfrac{k}{x+b} + c$ below, with y-intercept at $\left(0, \dfrac{10}{3}\right)$.



Which of the following is true?

(A) $k < b < c$

(B) $b < k < c$

(C) $k < c < b$

(D) $c < k < b$

8. The discrete random variable X has the following distribution:

X	0	1	2	3
$P(X = x)$	0.2	m	0.3	n

Given that $E(X) = 1.8$, find the values of m and n.

(A) $m = 0.15$ and $n = 0.35$

(B) $n = 0.15$ and $m = 0.35$

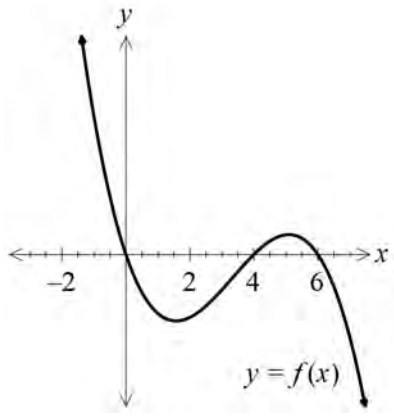
(C) $m = 0.5$ and $n = 0$

(D) $n = 0.5$ and $m = 0$

$0.3 = 2m$

$m = 0.15, n = 0.35$

9. Consider the graph of $y = f(x)$ below.



Which of the following integrals has the greatest value?

(A) $\int_{-2}^{6} f(x)\,dx$

(B) $\int_{0}^{6} f(x)\,dx$

(C) $\int_{-2}^{0} f(x)\,dx$

(D) $\int_{-2}^{4} f(x)\,dx$

10. Consider the cumulative probability density function

$$F(x) = \begin{cases} 0, & x < \dfrac{\pi}{2} \\ \cos^2 x, & \dfrac{\pi}{2} \le x \le \pi \\ 1, & x > \pi \end{cases}$$

What is the 3rd quartile of the distribution?

(A) $x = \dfrac{\pi}{6}$ (B) $x = \dfrac{5\pi}{8}$ (C) $x = \dfrac{3\pi}{4}$ (D) $x = \dfrac{5\pi}{6}$

End of Multiple Choice.

Mathematics Advanced

Section II Answer booklet

90 marks
Attempt Questions 11 – 35.
Allow about 2 hours and 45 minutes for this section.

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Please turn over

Question 11 (1 mark)

Factorise: $4x^2-(w+y)^2$ **1**

Question 12 (2 marks)

Consider the arithmetic sequence 16, 12, 8, ...

i) Find the 58^{th} term. **1**

ii) Find the sum of the first 35 terms. **1**

Question 13 (2 marks)

Given that $f(x) = 3x - 1$ and $g(x) = \dfrac{1}{2x}$ find:

i) $g(f(x))$. **1**

ii) The domain of $g(f(x))$. **1**

Question 14 (2 marks)

Differentiate with respect to x: $f(x) = \dfrac{3\sin x^2}{x}$ **2**

Question 15 (3 marks)

Oliver buys a car for $19000 and repays it over 4 years through equal monthly instalments. He pays a 20% deposit and interest is charged at 9% p.a. on the balance of the loan.
[Below is a table of present value interest factors for an annuity of $1]

Table of Present Value Interest Factors						
r	**0.0060**	**0.0065**	**0.0070**	**0.0075**	**0.0080**	**0.0085**
N						
45	39.33406	38.90738	38.48712	38.07318	37.66545	37.26383
46	40.09350	39.64965	39.21263	38.78231	38.35859	37.94133
47	40.84841	40.38714	39.93310	39.48617	39.04622	38.61311
48	41.59882	41.11986	40.64856	40.18478	39.72839	39.27924
49	42.34475	41.84785	41.35905	40.87820	40.40515	39.93975
50	43.08623	42.57113	42.06459	41.56645	41.07653	40.59470

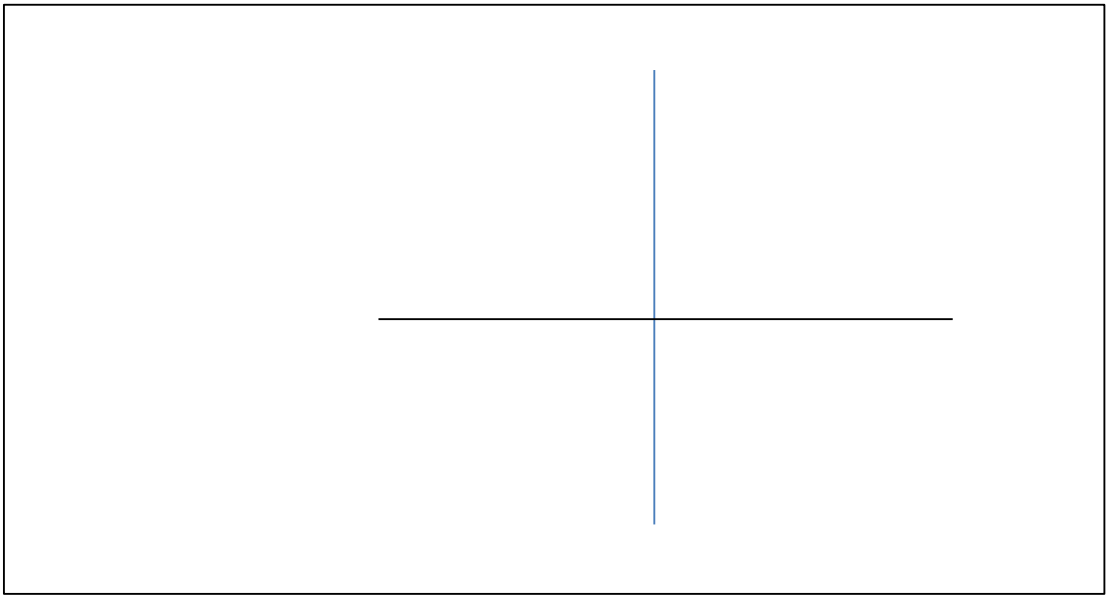
What is the total amount that Oliver pays for the car? **3**

Question 16 (2 marks)

Find $\int x(5-3x^2)^5\,dx$ **2**

Question 17 (3 marks)

(a) Sketch $y = |5 - x|$ **1**

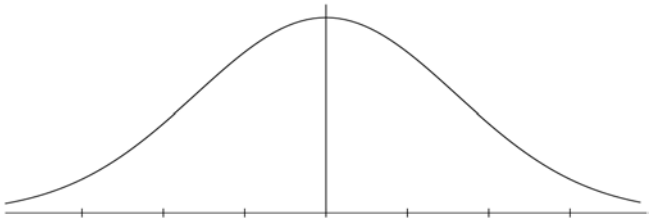


(b) Hence evaluate $\int_{-5}^{0} |5 - x| \, dx$ **2**

Question 18 (5 marks)

The working life of a particular brand of gumboot is normally distributed. The gumboots have a mean life of 5.4 years with a standard deviation of 1.2 years.

i) Complete the normal distribution curve with this information. **1**



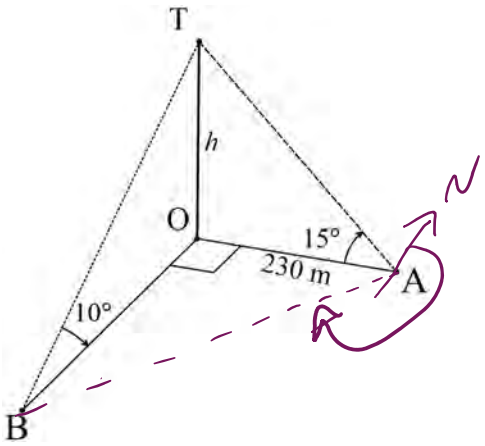
ii) What percentage of gumboots would have a life between 3 years and 6.6 years? **1**

iii) At the beginning of the Year 2022, 50 000 pairs of gumboots were produced. How many of these pairs of gumboots would you expect to last past the beginning of 2031? **1**

iv) A full refund is provided if a pair of gumboots lasts less than 2 years. Estimate the percentage of purchases that will be refunded? **2**
[A *z*-score table is provided on pages 37 – 38]

Question 19 (4 marks)

Anthony stands at point A, 230 m due east of the tower OT, of height *h* metres. The angle of elevation of T from A is 15º. Brian, at point B, is due south of the tower OT. The angle of elevation to the top of the tower from B is 10º.



i) Show that the height of the tower is 61.6 metres correct to 3 significant figures. **1**

ii) Find how far Brian is from the bottom of the tower. Round your answer to 3 significant figures. **1**

Question 19 continues on the next page.

Question 19 continued…

iii) What is the bearing of Brian from Anthony? **2**

Question 20 (2 marks)

Evaluate exactly $\int_1^2 \frac{3}{4x-1}dx$. **2**

Question 21 (3 marks)

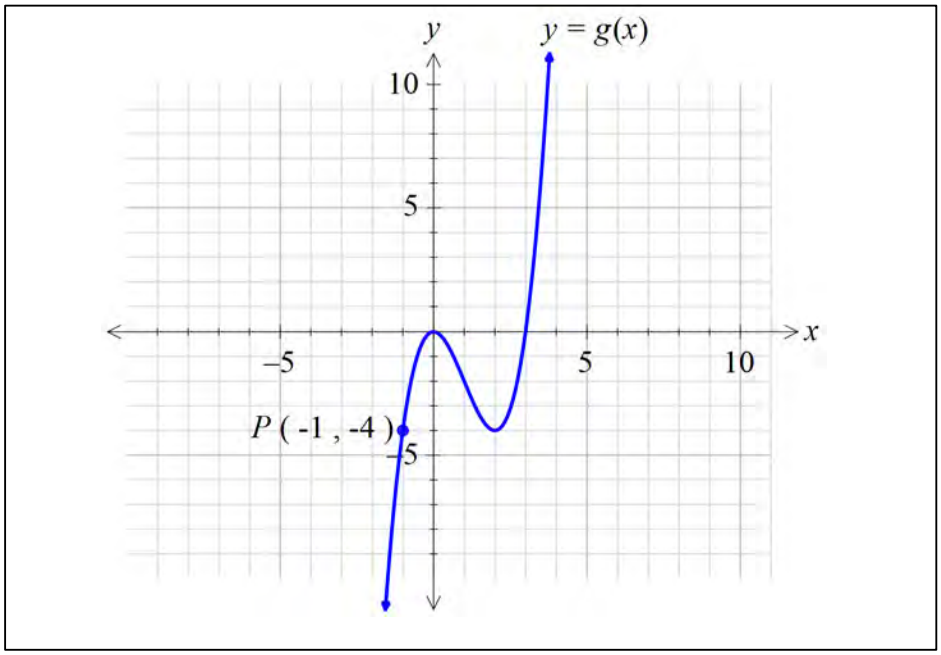
Find the exact value of the gradient of the tangent to $y = e^{2x}(x+2)$ at the point where $x = 1$.

3

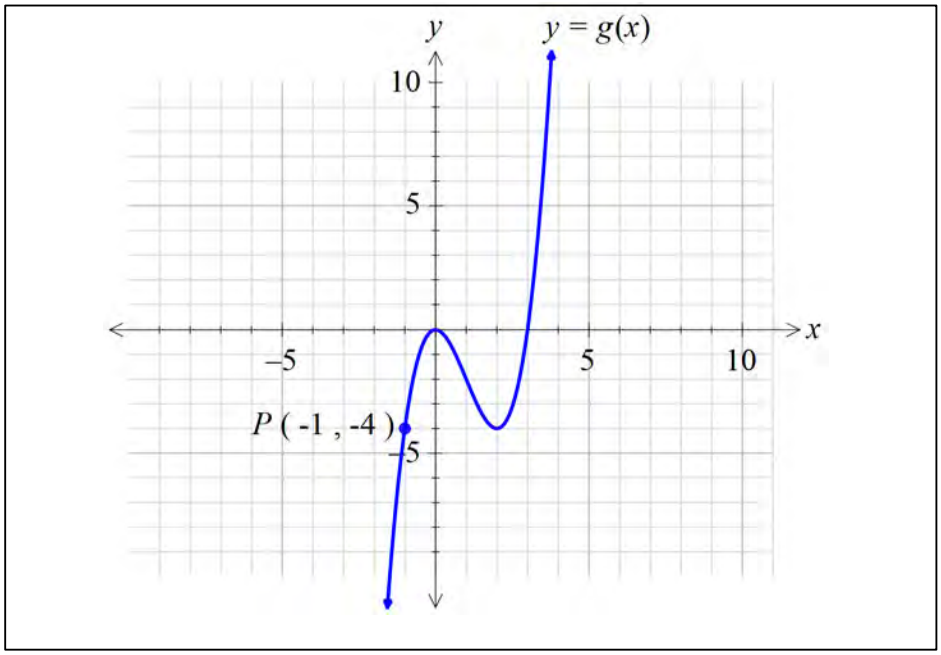
Question 22 (4 marks)

The function $y = g(x)$ is graphed below. On the sets of axes provided, sketch:

i) $y = g(x-2)+3$ **2**



ii) $y = 2g\left(-\frac{x}{2}\right)$ **2**



Question 23 (3 marks)

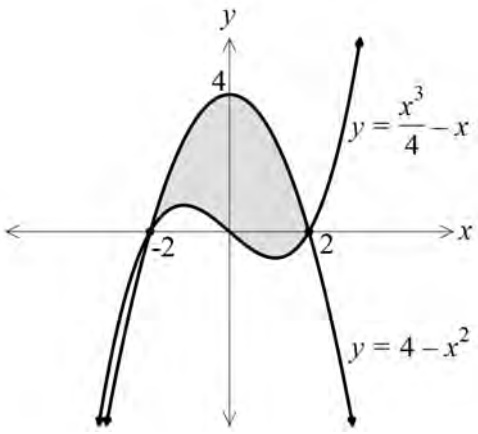
For what values of x is $f(x) = \frac{x^4}{6} - \frac{2x^3}{3} + 3x + 1$ concave down? **3**

Question 24 (2 marks)

The function $y = g(x)$ is transformed to the function $y = \frac{1}{2}g(3(x-1)) + 4$. Find the point $P(x, y)$ on $y = g(x)$, if its image point on the transformed function is $P'(-6, 5)$. **2**

Question 25 (3 marks)

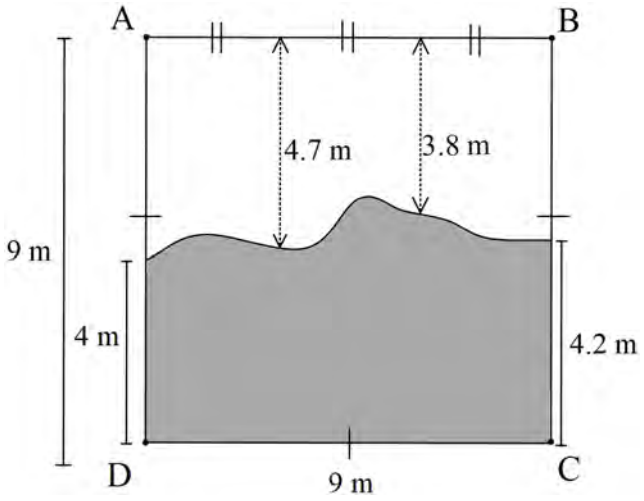
The curves $y = 4 - x^2$ and $y = \frac{x^3}{4} - x$ intersect at $x = 2$ and $x = -2$.



Determine the exact area of the shaded region. **3**

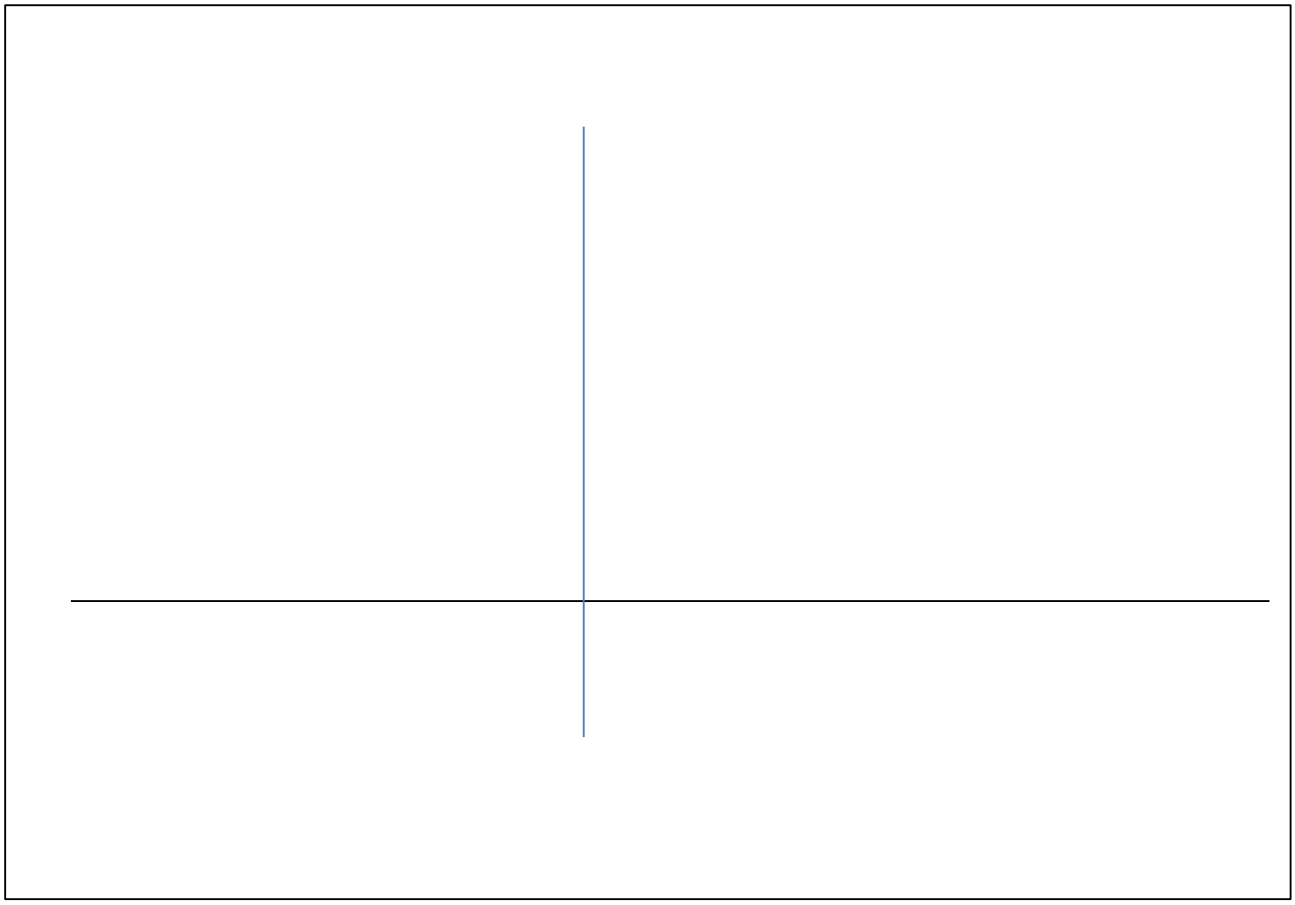
Question 26 (3 marks)

ABCD is a square. Approximate the shaded area by using the Trapezoidal Rule. **3**



Question 27 (6 marks)

(a) Sketch the graph of the curve $y = 3x^3 + 12x^2 + 4$ labelling the stationary points, point of inflection and the y-intercept. Do NOT determine the x-intercepts of the curve. (Use this page for your working. There is space for the sketch on the next page). **5**



(b) For what values of k does $3x^3 + 12x^2 + 4 = k$ have 2 solutions? **1**

Question 28 (4 marks)
A tennis ball is dropped from a height of 12 metres. After it first hits the ground, it rebounds to a height of 8 metres. It continues to rebound to a height of two thirds its previous rebound height.

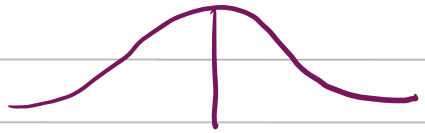
i) How high will the ball rise after the 6^{th} time it rebounds from the ground? **2**

ii) What is the total vertical distance that the ball will eventually travel? **2**

Question 29 (3 marks)

The blood pressure of women who are 50 years old is normally distributed with a mean of 124 and a standard deviation of 18.

Two 50-year-old women are chosen at random. What is the probability that the blood pressure of at least one woman is over 160? **3**



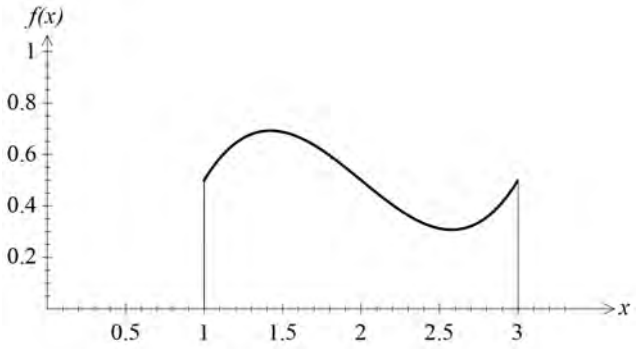
Question 30 (3 marks)

Sketch $y = 3\cos\left[2\left(x - \frac{\pi}{4}\right)\right]$ in the domain $[0, 2\pi]$. **3**

Question 31 (5 marks)

A continuous probability distribution is graphed below and is defined by:

$$f(x)=\begin{cases}\frac{1}{2}(x^3-6x^2+11x-5), & [1,3]\\ 0, & (-\infty,1)\cup(3,\infty)\end{cases}$$



i) Use the graph of $y=f(x)$ to **estimate** the mode of the distribution. **1**

i) Find the cumulative distribution function. **2**

ii) Find $P(X>2)$. **2**

Question 32 (5 marks)

Sam plays football, a sport where a team can Win, Lose or Draw.

Sam is prone to injury, and plays in 80% of her team's games.

When Sam plays, her team has a 73% chance of winning and a 7% chance of losing.
When Sam doesn't play her team has a 55% chance of winning and a 20% chance of losing.

i) By using a tree diagram or otherwise, show that the probability that Sam's team wins any given game is 69.4%. **2**

ii) Given that Sam's team doesn't lose in a particular match, what is the probability that Sam played in that match? **3**

Question 33 (7 marks)

Maryann invests $20000 into a superannuation fund at the **end of each year** to save for her retirement. The interest **compounds monthly** at a rate of 6 % p.a.

a) Show that after three years, the value of Maryann's superannuation fund is $63776.75. **3**

b) After 10 years, just after her 10^{th} deposit of $20000, Maryanne withdraws $100,000 to buy an apartment. Given that she continues to invest $20000 at the end of each year, find the value of Maryann's superannuation fund after a **further** 5 years? **4**

More space for Q33ii on next page.

Question 34 (6 marks)

The temperature $T(t)$ in ^{o}C during an average day in May in Katherine NT, can be modelled by the function $T(t) = 27.5 + 8.5\sin\left(\frac{\pi t}{12}\right)$, where *t* is the number of hours after 9am.

i) What is the maximum temperature in a day? **1**

ii) Sketch a graph for the function $T(t) = 27.5 + 8.5\sin\left(\frac{\pi t}{12}\right)$ from $0 \le t \le 24$. **2**

Question 34 continues on the next page.

Question 34 continued…

iii) Jimmy lives in Katherine, and his air conditioning unit is only set to be turned on if the outside temperature is above 33 ºC. Between what times during the day is his air conditioning on? Give your answer correct to the nearest minute. **3**

Question 35 (7 marks)

A car travelling at a constant speed of v km/h consumes fuel at the rate of $\frac{1}{20}\left(1+\frac{v^2}{60}\right)$ litres per hour.

Hai hires this car for a rental fee of $10 per hour. Fuel costs $2.12/litre and his total trip has a distance of 550 km.

i) Show that the total cost of the trip in dollars is given by $C = \frac{55583}{10v} + \frac{583v}{600}$. **3**

Question 35 continues on the next page.

Question 35 continued…

ii) Find the minimum cost Hai will have to pay for his trip. **4**

More space for Q35iib on next page.

END OF EXAMINATION.

Name: _____ **Teacher: MN MA GS LR RC SW**

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ASCHAM SCHOOL

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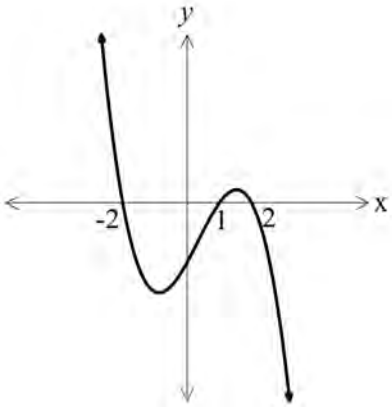

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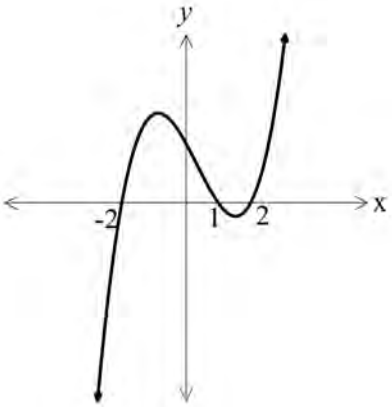
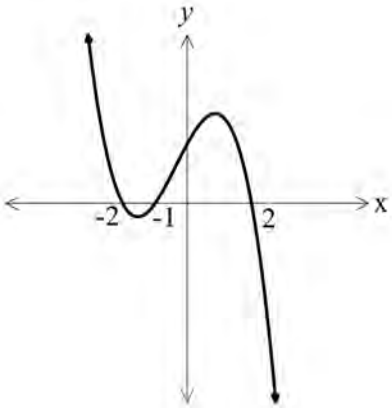
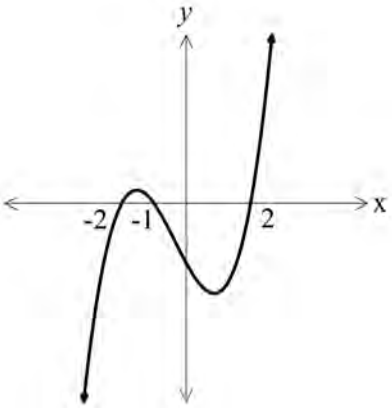


(C)

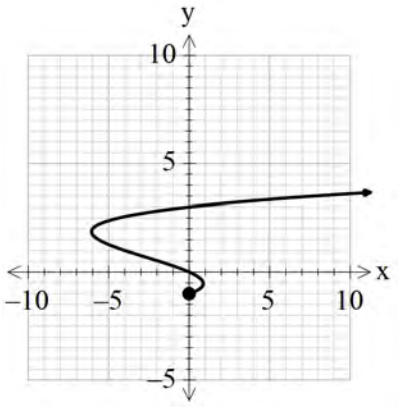


(D)



x int
x = -2, 2, 1

2. What type of relation is shown in the graph below?



(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

3. One card is selected at random from a pack of cards labelled 1 to 50. Find the probability that the card is an odd number or a number greater than 40.

(A) 0.1 (B) 0.58 (C) 0.6 (D) 0.7

$P(\text{odd} \cup >40) = \frac{1}{2} + \frac{10}{50} - \frac{5}{50}$

$=$

4 The function $y = f(x)$ is transformed to $y = f\left(\frac{1}{2}(x+2)\right)$.

$f\left(\frac{1}{2}(x+2)\right)$

Which of the following describes the transformations that took place?

$a = \frac{1}{2}$

$b = 2$

(A) Horizontal dilation factor of 2, followed by translation left 1 unit.

(B) Horizontal dilation factor of 2, followed by translation left 2 units.

(C) Horizontal dilation factor of $\frac{1}{2}$, followed by translation left 1 unit.

(D) Horizontal dilation factor of $\frac{1}{2}$, followed by translation left 2 units.

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(B) Period $= 6\pi$, Range: $[2, 6]$

(C) Period $= \frac{2\pi}{3}$, Range: $[-6, 6]$

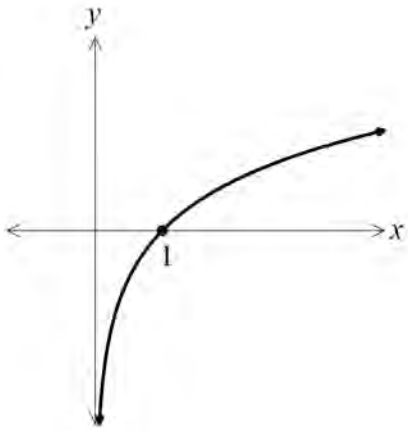
(D) Period $= \frac{2\pi}{3}$, Range: $[2, 6]$

Period $= \frac{2\pi}{1/3}$

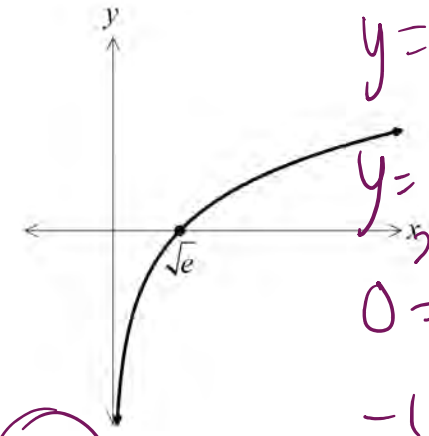
$= 6\pi$.

6. Which of the following could be the graph of the function $y = 2\log_e\left(\frac{1}{x}\right) + 1$

(A)



(B)



$y = 2\ln x^{-1} + 1$

$y = -2\ln x + 1$

x int

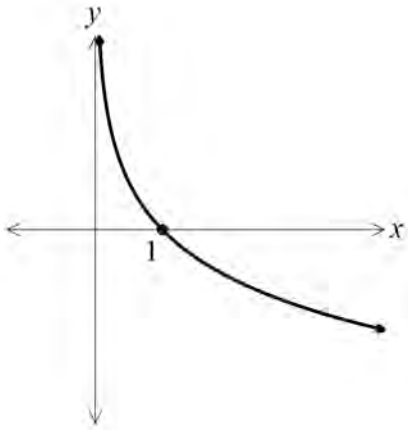
$0 = -2\ln x + 1$

$-1 = -2\ln x$

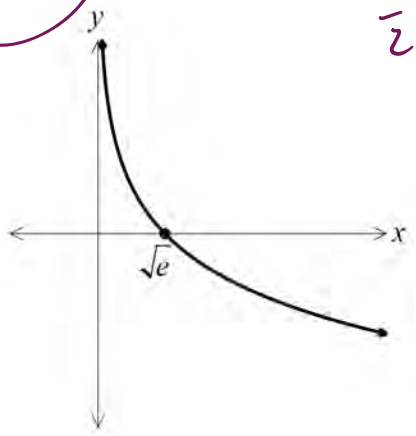
$\frac{1}{2} = \ln x$

$e^{1/2} = x$

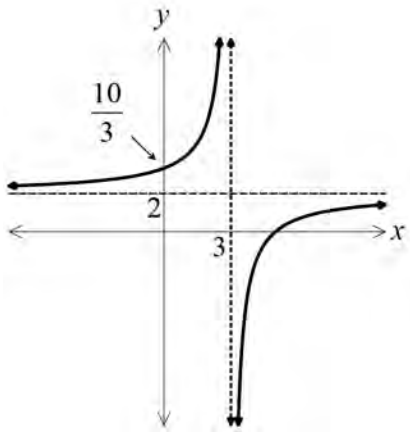
(C)



(D)



7. Consider the graph of $y = \dfrac{k}{x+b} + c$ below, with y-intercept at $\left(0, \dfrac{10}{3}\right)$.



$c = 2$

$b = -3$

$y = \dfrac{k}{x-3} + 2$

y int:

$\dfrac{10}{3} = \dfrac{k}{-3} + 2$

$-10 = k + -6$

$-4 = k$.

Which of the following is true?

(A) $k < b < c$

(B) $b < k < c$

(C) $k < c < b$

(D) $c < k < b$

8. The discrete random variable X has the following distribution:

X	0	1	2	3
$P(X = x)$	0.2	m	0.3	n

Given that $E(X) = 1.8$, find the values of m and n.

(A) $m = 0.15$ and $n = 0.35$

(B) $n = 0.15$ and $m = 0.35$

(C) $m = 0.5$ and $n = 0$

(D) $n = 0.5$ and $m = 0$

$m + n + 0.5 = 1$

$m + n = 0.5$

$n = 0.5 - m$

$m + 0.6 + 3n = 1.8$

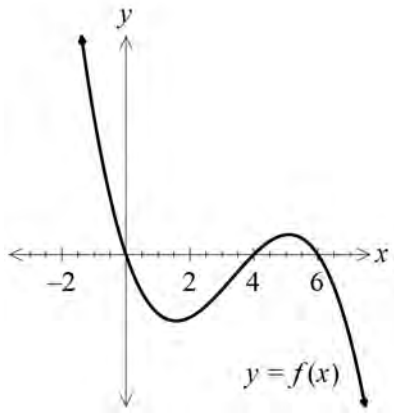
$m + 0.6 + 3(0.5 - m) = 1.8$

$m + 0.6 + 1.5 - 3m = 1.8$

$0.3 = 2m$

$m = 0.15, n = 0.35$

9. Consider the graph of $y = f(x)$ below.



Which of the following integrals has the greatest value?

(A) $\int_{-2}^{6} f(x)\,dx$

(B) $\int_{0}^{6} f(x)\,dx$

(C) $\int_{-2}^{0} f(x)\,dx$

(D) $\int_{-2}^{4} f(x)\,dx$

10. Consider the cumulative probability density function

$$F(x) = \begin{cases} 0, & x < \frac{\pi}{2} \\ \cos^2 x, & \frac{\pi}{2} \le x \le \pi \\ 1, & x > \pi \end{cases}$$

What is the 3rd quartile of the distribution?

(A) $x = \frac{\pi}{6}$ (B) $x = \frac{5\pi}{8}$ (C) $x = \frac{3\pi}{4}$ (D) $x = \frac{5\pi}{6}$

End of Multiple Choice.

Mathematics Advanced

Section II Answer booklet

90 marks
Attempt Questions 11 – 35.
Allow about 2 hours and 45 minutes for this section.

Instructions:

- Answer each question in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Additional writing space is available on pages 39 to 43. If you use this space. Place start a new sheet of paper for each question.

Please turn over

Question 11 (1 mark)

Factorise: $4x^2-(w+y)^2$ **1**

$4x^2-(w+y)^2 = (2x)^2-(w+y)^2$

$=(2x-(w+y))(2x+(w+y))$

$=(2x-w-y)(2x+w+y)$

Question 12 (2 marks)

Consider the arithmetic sequence 16, 12, 8, ...

i) Find the 58th term. **1**

$T_n = a+(n-1)d$

$T_{58} = 16+(58-1)(-4)$

$= -212$

$a=16$

$d=-4$

$n=58$

ii) Find the sum of the first 35 terms. **1**

$S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{35} = \frac{35}{2}(2(16)+(35-1)(-4))$

$= -1820$

$a=16$

$d=-4$

$n=35$

Question 13 (2 marks)

Given that $f(x) = 3x - 1$ and $g(x) = \frac{1}{2x}$ find:

i) $g(f(x))$. **1**

$g(f(x)) = \frac{1}{2(3x-1)}$

$= \frac{1}{6x-2}$

ii) The domain of $g(f(x))$. **1**

$6x - 2 \neq 0$

$6x \neq 2$

$x \neq \frac{1}{3}$

$\therefore$ Domain: $x \in (-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

Question 14 (2 marks)

Differentiate with respect to x: $f(x) = \frac{3\sin x^2}{x}$ **2**

$y' = \frac{u'v - v'u}{v^2}$

$= \frac{(6x\cos x^2)(x) - (1)(3\sin x^2)}{x^2}$

$= \frac{6x^2\cos x^2 - 3\sin x^2}{x^2}$

$u = 3\sin x^2$

$u' = 6x\cos x^2$

$v = x$

$v' = 1$

Question 15 (3 marks)

Oliver buys a car for $19000 and repays it over 4 years through equal monthly instalments. He pays a 20% deposit and interest is charged at 9% p.a. on the balance of the loan.
[Below is a table of present value interest factors for an annuity of $1]

Table of Present Value Interest Factors						
r	0.0060	0.0065	0.0070	0.0075	0.0080	0.0085
N						
45	39.33406	38.90738	38.48712	38.07318	37.66545	37.26383
46	40.09350	39.64965	39.21263	38.78231	38.35859	37.94133
47	40.84841	40.38714	39.93310	39.48617	39.04622	38.61311
48	41.59882	41.11986	40.64856	40.18478	39.72839	39.27924
49	42.34475	41.84785	41.35905	40.87820	40.40515	39.93975
50	43.08623	42.57113	42.06459	41.56645	41.07653	40.59470

What is the total amount that Oliver pays for the car? **3**

$PV = mx$

$m = \frac{15200}{40.18478}$

$m = 378.25$ (nearest cent)

$n = 4 \times 12 = 48$

$r = \frac{9\%}{12} = 0.0075$

$PV = 0.8 \times 19000 = 15200$

$\text{Total} = 378.25 \times 48 + 0.2 \times 19000$

$= \$21956.$

[$= \$21956.13$ (nearest cent) if used exact m]

Question 16 (2 marks)

Find $\int x(5-3x^2)^5\,dx$ **2**

$-\frac{1}{6}\int 6x(5-3x^2)^5\,dx$

$= -\frac{1}{6}\left[\frac{(5-3x^2)^6}{6}\right] + C$

$= -\frac{(5-3x^2)^6}{36} + C$

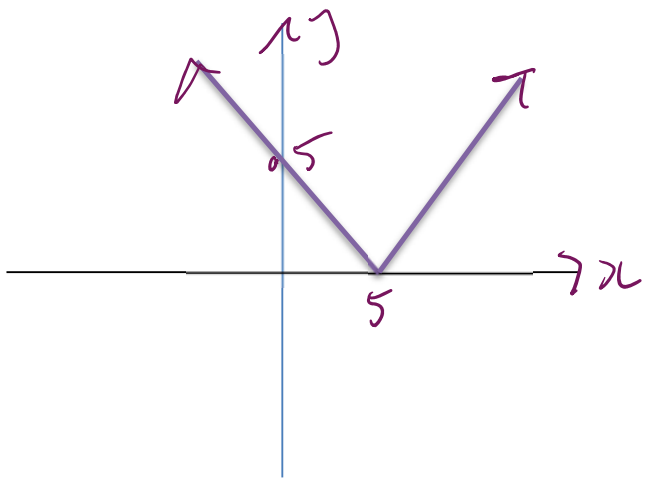
$f(x) = 5-3x^2$

$f'(x) = -6x$

Question 17 (3 marks)

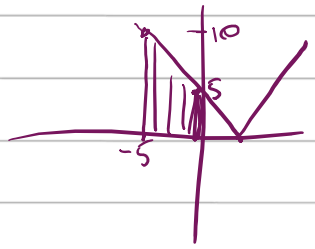
(a) Sketch $y = |5 - x|$ **1**

x int.
$0 = 5 - x$
$x = 5$



(b) Hence evaluate $\int_{-5}^{0} |5 - x|\, dx$ **2**

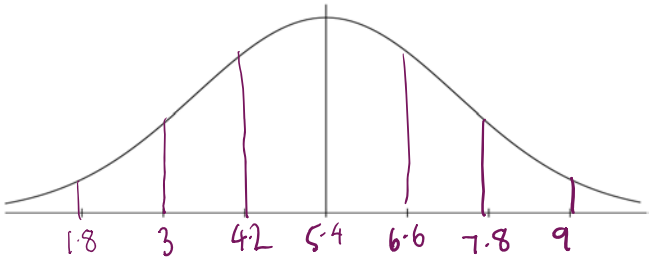
$$\int_{-5}^{0} |5-x|\, dx = \frac{5}{2}(5+10)$$
$$= 37.5$$



Question 18 (5 marks)

The working life of a particular brand of gumboot is normally distributed. The gumboots have a mean life of 5.4 years with a standard deviation of 1.2 years.

i) Complete the normal distribution curve with this information. **1**



ii) What percentage of gumboots would have a life between 3 years and 6.6 years? **1**

$\frac{95\cdot1\%}{2} + \frac{68\%}{2} = 81\cdot5\%$

iii) At the beginning of the Year 2022, 50 000 pairs of gumboots were produced. How many of these pairs of gumboots would you expect to last past the beginning of 2031? **1**

$50000 \times \frac{1-0\cdot99\cdot7}{2} = 75$

$\therefore$ 75 pairs.

iv) A full refund is provided if a pair of gumboots lasts less than 2 years. Estimate the percentage of purchases that will be refunded? **2**
[A z-score table is provided on pages 37 – 38]

$x = 2$
$\mu = 5\cdot4$
$\sigma = 1\cdot2$

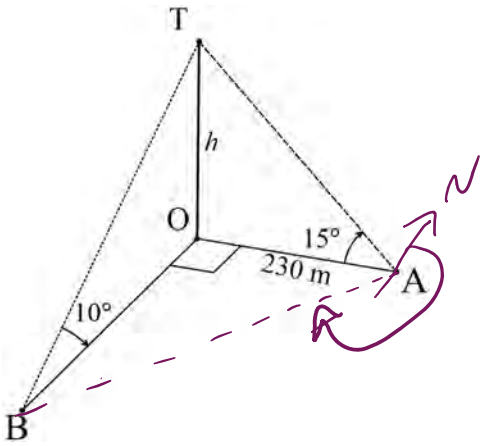
$\therefore z = \frac{x-\mu}{\sigma}$
$= \frac{2-5\cdot4}{1\cdot2}$
$\therefore z = 2\cdot833\ldots$
$-2\cdot83$ (2dp)

$P(z < -2\cdot83) = 0\cdot23\%$

$\therefore 0\cdot23\%$ could be refunded

Question 19 (4 marks)

Anthony stands at point A, 230 m due east of the tower OT, of height *h* metres. The angle of elevation of T from A is 15°. Brian, at point B, is due south of the tower OT. The angle of elevation to the top of the tower from B is 10°.



i) Show that the height of the tower is 61.6 metres correct to 3 significant figures. **1**

$\tan\theta = \frac{O}{A}$

$\tan 15 = \frac{h}{230}$

$h = 230 \times \tan 15$

$= 61.6283\ldots$

$= 61.6\text{m}$ (3sf)

ii) Find how far Brian is from the bottom of the tower. Round your answer to 3 significant figures. **1**

$\tan 10° = \frac{h}{OB}$

$OB = \frac{61.6}{\tan 10°}$

$OB = 349.3509\ldots$

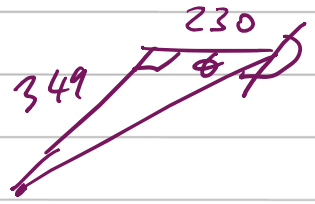
$= 349\text{m}$ (3sf.)

(or 350m if using exact part i answer).

Question 19 continues on the next page.

Question 19 continued…

iii) What is the bearing of Brian from Anthony? **2**



let $\angle OAB = \theta$

$\tan\theta = \frac{349}{230}$

$\theta = 56.614\ldots^\circ$

$\therefore$ Bearing $= 270 - 56.614\ldots$

$\simeq 213.38\ldots$

$= 213^\circ$ (nearest degree)

Question 20 (2 marks)

Evaluate exactly $\int_1^2 \frac{3}{4x-1}dx$. **2**

$$\int_1^2 \frac{3}{4x-1}dx = \frac{3}{4}\int_1^2 \frac{1\times 4}{4x-1}dx \qquad \begin{vmatrix} f(x)=4x-1 \\ f'(x)=4 \end{vmatrix}$$

$$= \frac{3}{4}\left[\ln|4x-1|\right]_1^2$$

$$= \frac{3}{4}\left[\ln|7| - \ln|3|\right]$$

$$= \frac{3}{4}\ln\left|\frac{7}{3}\right|$$

Question 21 (3 marks)

Find the exact value of the gradient of the tangent to $y = e^{2x}(x+2)$ at the point where $x=1$.

3

$$y = e^{2x}(x+2) \qquad u = e^{2x} \quad v = x+2$$

$$u' = 2e^{2x} \quad v' = 1$$

$$y' = 2e^{2x}(x+2) + 1(e^{2x})$$

$$= e^{2x}(2(x+2)+1)$$

$$= e^{2x}(2x+5)$$

at $x=1$

$$m_T = e^2(2(1)+5)$$

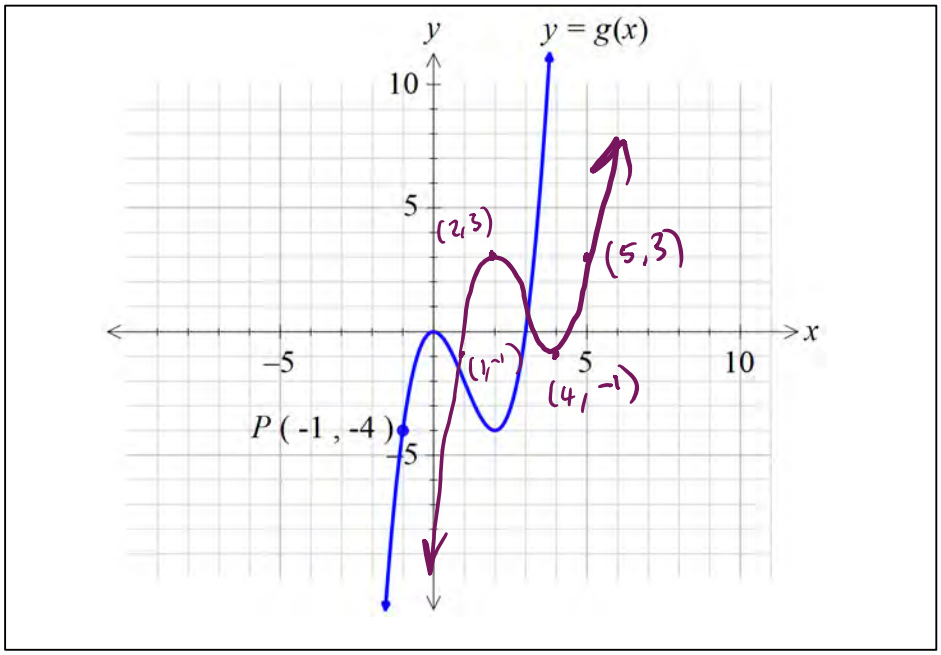
$$= 7e^2$$

Question 22 (4 marks)

The function $y = g(x)$ is graphed below. On the sets of axes provided, sketch:

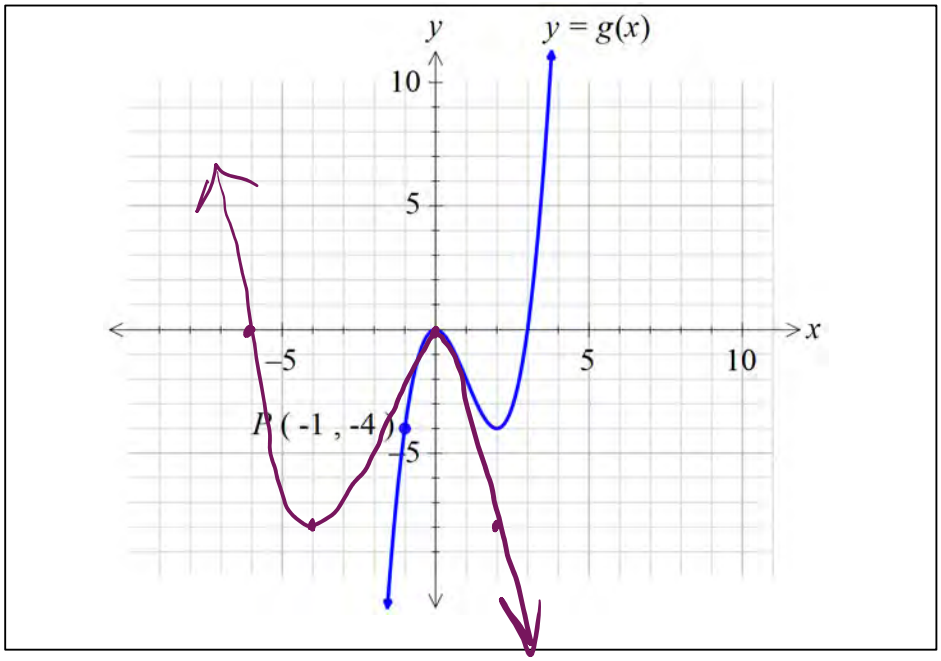
i) $y = g(x-2)+3$ **2**

$b=-2$
$c=3$



ii) $y = 2g\left(-\frac{x}{2}\right)$ **2**

$a=-\frac{1}{2}$ ↳ $x\times -2$
$k=2$. ↳ $y\times 2$



$(0,0)\to(0,0)$
$(3,0)\to(-6,0)$
$(2,-4)\to(-4,-8)$
$(-1,-4)\to(2,-8)$

Question 23 (3 marks)

For what values of x is $f(x)=\frac{x^4}{6}-\frac{2x^3}{3}+3x+1$ concave down? **3**

$f'(x)=\frac{4x^3}{6}-\frac{6x^2}{3}+3$

$=\frac{2x^3}{3}-2x^2+3$

$f''(x)=\frac{6x^2}{3}-4x$

$=2x^2-4x$

$f''(x)<0$

$2x^2-4x<0$

$2x(x-2)<0$

$\therefore 0<x<2$

Question 24 (2 marks)

The function $y=g(x)$ is transformed to the function $y=\frac{1}{2}g(3(x-1))+4$. Find the point $P(x,y)$ on $y=g(x)$, if its image point on the transformed function is $P'(-6,5)$. **2**

$a=3$

$b=-1$

$k=\frac{1}{2}$

$c=4$

$\frac{x}{3}+1=-6$

$x=-7$

$x=-21$

$\frac{y}{2}+4=5$

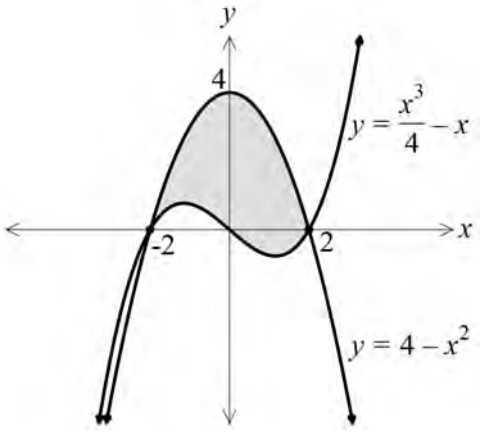
$\frac{y}{2}=1$

$y=2$

$\therefore P(-21,2)$

Question 25 (3 marks)

The curves $y = 4 - x^2$ and $y = \frac{x^3}{4} - x$ intersect at $x = 2$ and $x = -2$.



Determine the exact area of the shaded region. **3**

$$A = \int_{-2}^{2} (4 - x^2) - \left(\frac{x^3}{4} - x\right) dx$$

$$= \int_{-2}^{2} 4 - x^2 - \frac{x^3}{4} + x \, dx$$

$$= \left[4x - \frac{x^3}{3} - \frac{x^4}{16} + \frac{x^2}{2} \right]_{-2}^{2}$$

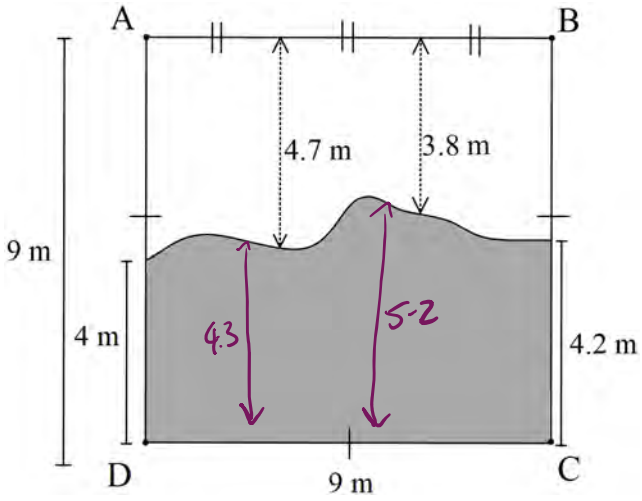
$$= \left[4(2) - \frac{2^3}{3} - \frac{2^4}{16} + \frac{2^2}{2} \right] - \left[4(-2) - \frac{(-2)^3}{3} - \frac{(-2)^4}{16} + \frac{(-2)^2}{2} \right]$$

$$= \frac{19}{3} - -\frac{13}{3}$$

$$= \frac{32}{3} u^2$$

Question 26 (3 marks)

ABCD is a square. Approximate the shaded area by using the Trapezoidal Rule. **3**



$A \doteqdot \frac{b-a}{2n}\left[f(a)+f(b)+2\left(f(x_1)+\dots f(x_{n-1})\right)\right]$

$\doteqdot \frac{9-0}{2\times 3}\left(4+4.2+2\left(4.3+5.2\right)\right)$

$\doteqdot 40.8u^2$

Question 27 (6 marks)

(a) Sketch the graph of the curve $y = 3x^3 + 12x^2 + 4$ labelling the stationary points, point of inflection and the y-intercept. Do NOT determine the x-intercepts of the curve. (Use this page for your working. There is space for the sketch on the next page). **5**

Stationary points when $y' = 0$

$y = 3x^3 + 12x^2 + 4$

$y' = 9x^2 + 24x$

$y'' = 18x + 24$.

$9x^2 + 24x = 0$

$3x(3x + 8) = 0$

$x = 0, -\frac{8}{3}$.

at $x = 0$, $y = 4 \rightarrow (0, 4)$

at $x = -\frac{8}{3}$, $y = \frac{292}{9} \rightarrow \left(-\frac{8}{3}, \frac{292}{9}\right)$

Check Nature:

at $x = 0$

$y'' = 18(0) + 24$

$= 24$

> 0 ∪

at $x = -\frac{8}{3}$

$y'' = 18\left(-\frac{8}{3}\right) + 24$

$= -24$

< 0 ∩

∴ $(0, 4)$ is a minimum turning point and $\left(-\frac{8}{3}, \frac{292}{9}\right)$ is a maximum turning point.

Point of Inflection:

$y'' = 0$

$18x + 24 = 0$

$x = -\frac{24}{18}$

$x = -\frac{4}{3}$

at $x = -\frac{4}{3}$

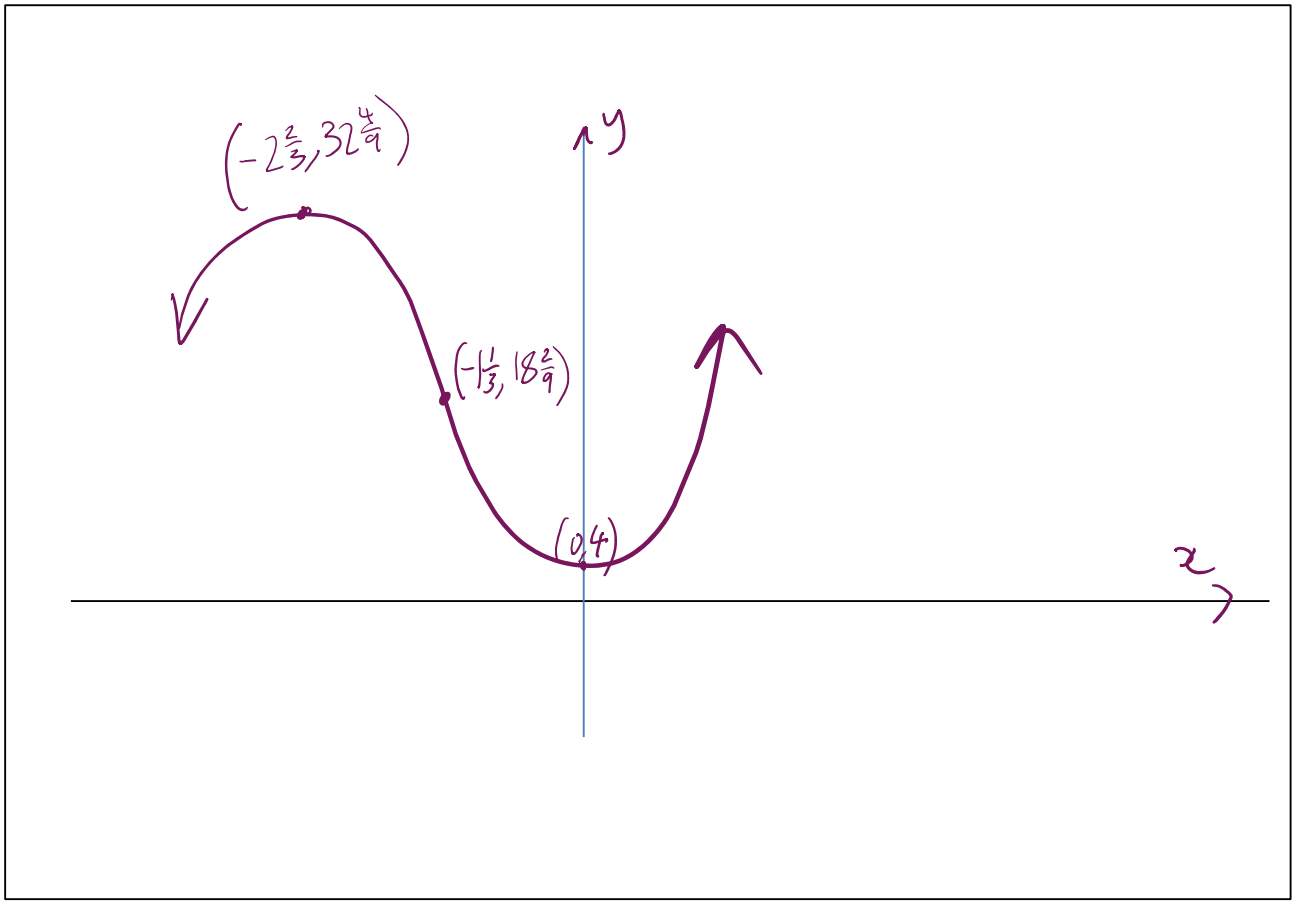
$y = \frac{164}{9}$

∴ $\left(-\frac{4}{3}, \frac{164}{9}\right)$

check:

x	-2	$-\frac{4}{3}$	-1
y''	-12	0	6
	∩		∪

∴ $\left(-\frac{4}{3}, \frac{164}{9}\right)$ is a Non horizontal point of Inflection

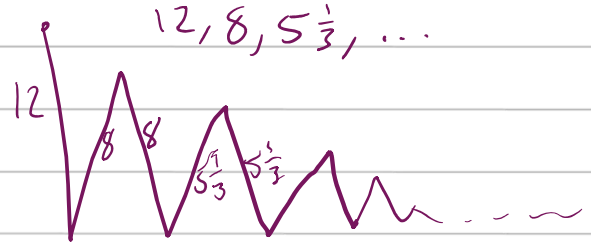


(b) For what values of k does $3x^3 + 12x^2 + 4 = k$ have 2 solutions? **1**

$k = 4$, $k = 32\frac{4}{9}$.

Question 28 (4 marks)
A tennis ball is dropped from a height of 12 metres. After it first hits the ground, it rebounds to a height of 8 metres. It continues to rebound to a height of two thirds its previous rebound height.

i) How high will the ball rise after the 6th time it rebounds from the ground? **2**



$12, 8, 5\frac{1}{3}, \ldots$

$r = \frac{2}{3}$

let $a = 8$

$\therefore n = 6$

$T_n = ar^{n-1}$

$= 8\left(\frac{2}{3}\right)^{6-1}$

$= 1.0534\ldots$

$= 1.05\text{ m (2dp)}$

$\therefore$ 6th Bounce is 1.05m high.

ii) What is the total vertical distance that the ball will eventually travel? **2**

$S = 12 + 8 + 8 + 5\frac{1}{3} + 5\frac{1}{3} + 3\frac{5}{9} + 3\frac{5}{9} + \ldots$

$= 12 + 2\left(8 + 5\frac{1}{3} + 3\frac{5}{9} \ldots\right)$

$= 12 + 2\left(\frac{8}{1 - \frac{2}{3}}\right)$

$= 60\text{ m}$

$\therefore$ will travel 60m.

Question 29 (3 marks)

The blood pressure of women who are 50 years old is normally distributed with a mean of 124 and a standard deviation of 18.

Two 50-year-old women are chosen at random. What is the probability that the blood pressure of at least one woman is over 160? **3**

$P(X>160) = \frac{100\% - 95\%}{2}$

$= 2.5\%$

$\therefore P(\text{at least one} > 160) = 1 - P(\text{Both less than } 160)$

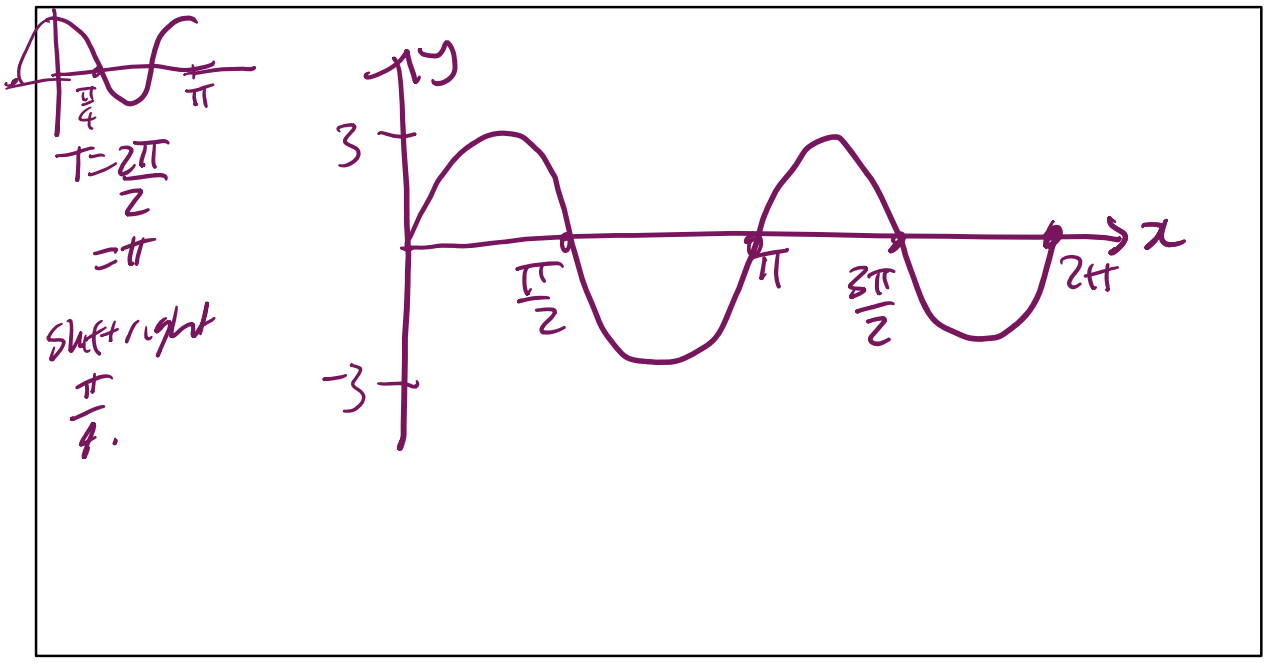
$= 1 - (97.5\% \times 97.5\%)$

$= 0.049375$

$= 4.9\% \text{ (1dp)}.$

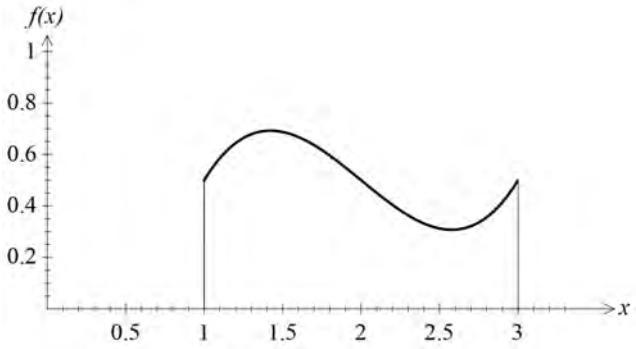
Question 30 (3 marks)

Sketch $y = 3\cos\left[2\left(x - \frac{\pi}{4}\right)\right]$ in the domain $[0, 2\pi]$. **3**



Question 31 (5 marks)
A continuous probability distribution is graphed below and is defined by:

$$f(x)=\begin{cases}\frac{1}{2}(x^3-6x^2+11x-5), & [1,3]\\ 0, & (-\infty,1)\cup(3,\infty)\end{cases}$$



i) Use the graph of $y=f(x)$ to **estimate** the mode of the distribution. **1**

Mode: $x=1.4$

i) Find the cumulative distribution function. **2**

$$F(x)=\int_1^x \frac{1}{2}(x^3-6x^2+11x-5)\,dx$$

$$=\frac{1}{2}\left[\frac{x^4}{4}-2x^3+\frac{11x^2}{2}-5x\right]_1^x$$

$$=\frac{1}{2}\left[\frac{x^4}{4}-2x^3+\frac{11x^2}{2}-5x-\left(\frac{1}{4}-2+\frac{11}{2}-5\right)\right]$$

$$=\frac{1}{2}\left[\frac{x^4}{4}-2x^3+\frac{11x^2}{2}-5x-\left(-\frac{5}{4}\right)\right]$$

$$=\frac{1}{8}\left[x^4-8x^3+22x^2-20x+5\right]$$

ii) Find $P(X>2)$. **2**

$$P(X>2)=1-P(x<2)$$

$$=1-F(2)$$

$$=1-\left[\frac{1}{8}\left(2^4-8(2)^3+22(2)^2-20(2)+5\right)\right]$$

$$=\frac{3}{8}$$

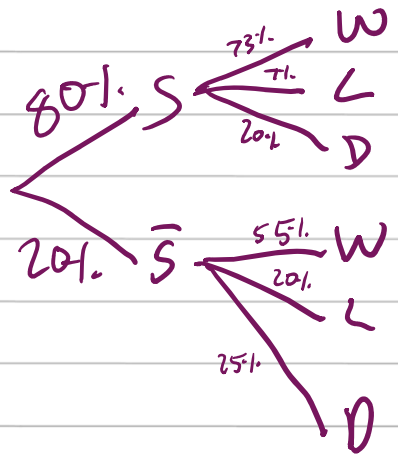
Question 32 (5 marks)

Sam plays football, a sport where a team can Win, Lose or Draw.

Sam is prone to injury, and plays in 80% of her team's games.

When Sam plays, her team has a 73% chance of winning and a 7% chance of losing.
When Sam doesn't play her team has a 55% chance of winning and a 20% chance of losing.

i) By using a tree diagram or otherwise, show that the probability that Sam's team wins any given game is 69.4%. **2**



$P(W) = P(SW) + P(\bar{S}W)$
$= (0.8 \times 0.73) + (0.2 \times 0.55)$
$= 0.584 + 0.11$
$= 0.694$
$= 69.4\%$

ii) Given that Sam's team doesn't lose in a particular match, what is the probability that Sam played in that match? **3**

$$P(S|\bar{L}) = \frac{P(S \cap \bar{L})}{P(\bar{L})}$$
$$= \frac{(0.8 \times 0.73) + (0.8 \times 0.2)}{1 - [0.8 \times 0.07 + 0.2 \times 0.2]}$$
$$= \frac{0.744}{0.904}$$
$$= 82.3\% \text{ (1dp)}$$

Question 33 (7 marks)

Maryann invests $20000 into a superannuation fund at the **end of each year** to save for her retirement. The interest **compounds monthly** at a rate of 6 % p.a.

a) Show that after three years, the value of Maryann's superannuation fund is $63776.75. **3**

Let A_n be the account balance after n months $r = 6\% \div 12$
$= 0.005$

$A_{12} = 20000$

$A_{24} = 20000(1.005)^{12} + 20000$

$A_{36} = [20000(1.005)^{12} + 20000](1.005)^{12} + 20000$

$= 20000(1.005)^{24} + 20000(1.005)^{12} + 20000$

$= \$63776.7517...$

$= \$63776.75$ (nearest cent)

b) After 10 years, just after her 10th deposit of $20000, Maryanne withdraws $100,000 to buy an apartment. Given that she continues to invest $20000 at the end of each year, find the value of Maryann's superannuation fund after a **further** 5 years? **4**

From (i) $A_3 = 20000(1.005)^{24} + 20000(1.005)^{12} + 20000$

$= 20000(1 + 1.005^{12} + 1.005^{24})$

$\therefore A_{120} = 20000(1 + 1.005^{12} + 1.005^{24} + \dots 1.005^{108})$

$= 20000\left(1 \times \left[\frac{(1.005^{12})^{10} - 1}{1.005^{12} - 1}\right]\right)$

$= \$265702.271$

$= \$265702.27$ (nearest cent)

More space for Q33ii on next page.

After 10 years & after withdrawal:

Value = $165702.27.

After 5 more years. (60 months)

$$\text{Value} = (165702.27)(1.005)^{60} + \text{New Annuity}.$$

New Annuity.

$$A_{60} = 20000\left(1 + 1.005^{12} + 1.005^{24} + \dots + 1.005^{48}\right)$$

$$= 20000\left(\frac{(1.005)^{12\times 5} - 1}{1.005^{12} - 1}\right)$$

$$= \$113120.14555.$$

$\therefore$ Value after 5 more Years:

$$= 165702.27(1.005)^{60} + 113120.14455$$

$$= \$336627.6767\ldots$$

$$= \$336627.68 \text{ (nearest cent)}.$$

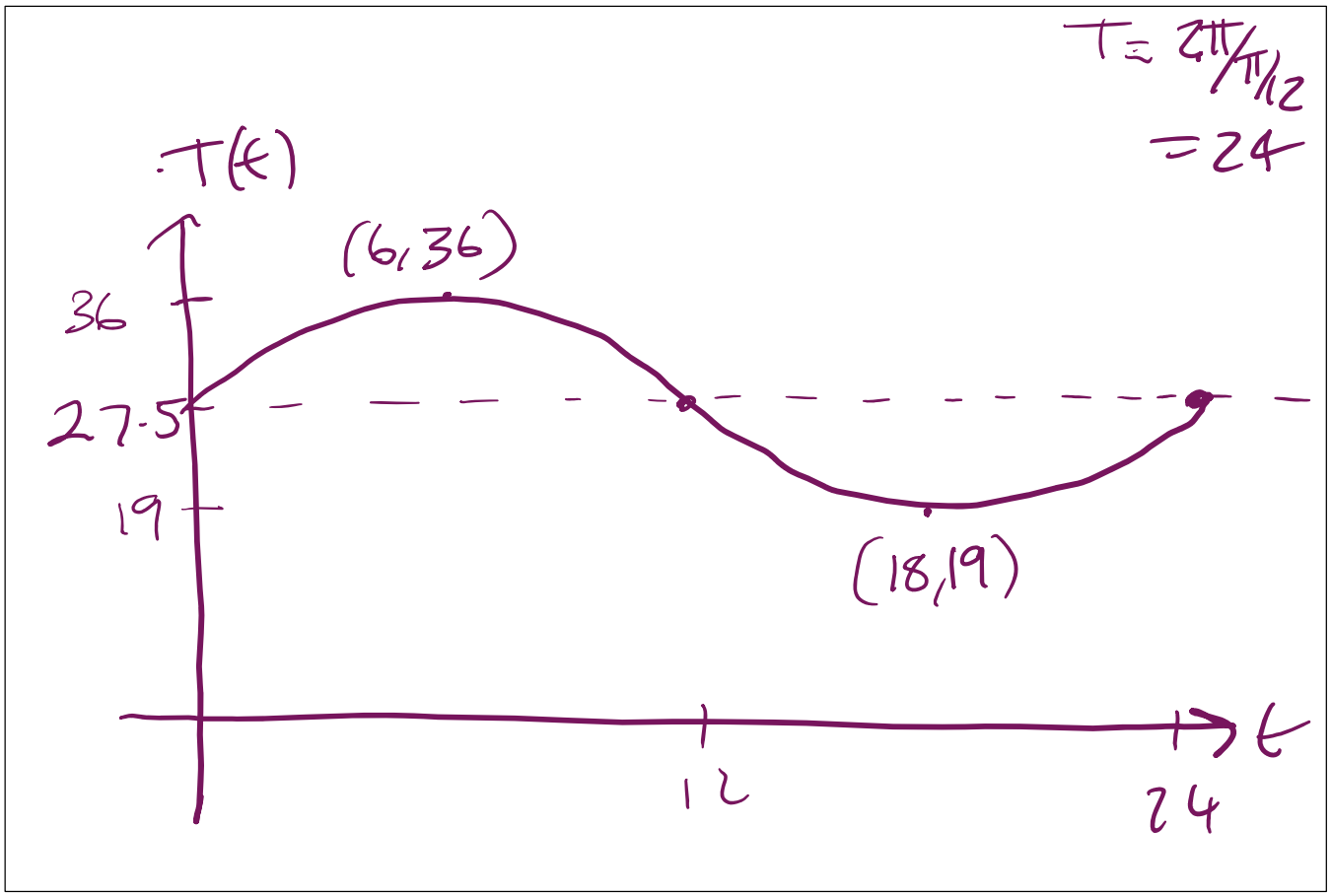
Question 34 (6 marks)

The temperature $T(t)$ in $^{\circ}C$ during an average day in May in Katherine NT, can be modelled by the function $T(t) = 27.5 + 8.5\sin\left(\frac{\pi t}{12}\right)$, where t is the number of hours after 9am.

i) What is the maximum temperature in a day? **1**

Max temp = 27.5 + 8.5(1)
= 36°C

ii) Sketch a graph for the function $T(t) = 27.5 + 8.5\sin\left(\frac{\pi t}{12}\right)$ from $0 \le t \le 24$. **2**



Question 34 continues on the next page.

Question 34 continued…

iii) Jimmy lives in Katherine, and his air conditioning unit is only set to be turned on if the outside temperature is above 33 °C. Between what times during the day is his air conditioning on? Give your answer correct to the nearest minute. **3**

$33 = 27.5 + 8.5\sin\left(\frac{\pi t}{12}\right)$

$\frac{11}{17} = \sin\left(\frac{\pi t}{12}\right)$ ✓ ✓

$\frac{\pi t}{12} = 0.7037\ldots, 2.4378\ldots$ ref $\angle = 0.7037\ldots$

$t = 2.688\ldots, 9.3119\ldots$

$= 2°41', 9°19'$ (nearest minute)

∴ Airconditioning is on between:

11:41am and 6:19pm.

Question 35 (7 marks)

A car travelling at a constant speed of v km/h consumes fuel at the rate of $\frac{1}{20}\left(1+\frac{v^2}{60}\right)$ litres per hour.

Hai hires this car for a rental fee of \$10 per hour. Fuel costs \$2.12/litre and his total trip has a distance of 550 km.

i) Show that the total cost of the trip in dollars is given by $C=\frac{55583}{10v}+\frac{583v}{600}$. **3**

Cost = $10 \times t + 2.12 \times \left[\frac{1}{20}\left(1+\frac{v^2}{60}\right)\right] \times t$

$t = \frac{d}{v}$

$t = \frac{550}{v}$

$C = 10 \times \frac{550}{v} + 2.12 \times \left(\frac{1}{20} \times \left(1+\frac{v^2}{60}\right)\right) \times \frac{550}{v}$

$= \frac{5500}{v} + \frac{583}{10v}\left(1+\frac{v^2}{60}\right)$

$= \frac{5500}{v} + \frac{583}{10v} + \frac{583v^2}{600v}$

$= \frac{55583}{10v} + \frac{583v}{600}$

Question 35 continues on the next page.

Question 35 continued…

ii) Find the minimum cost Hai will have to pay for his trip. **4**

$$C = \frac{55583}{10}V^{-1} + \frac{583V}{600}$$

$$\frac{dC}{dV} = -\frac{55583}{10}V^{-2} + \frac{583}{600}$$

$$\frac{dC}{dV} = 0$$

$$-\frac{55583}{10V^2} + \frac{583}{600} = 0$$

$$\frac{583}{600} = \frac{55583}{10V^2}$$

$$10V^2 \times 583 = 55583 \times 600$$

$$V^2 = \frac{55583 \times 600}{10 \times 583}$$

$$V^2 = 5720.377\ldots$$

$$V = \pm\sqrt{5720.377\ldots}$$

$$V = \pm 75.633\ldots \text{ km/h}$$

$$V > 0$$

$$V = 75.633\ldots \text{ km/h}$$

More space for Q35iib on next page.

Check Nature:

$$C' = -\frac{55583}{10}v^{-2} + \frac{583}{600}$$

$$C'' = \frac{11166}{10}v^{-3}$$

$$= \frac{11166}{10v^3}$$

> 0 for all $v > 0$ ✓

$\therefore v = 75.633\ldots$ km/h gives minimum cost.

$$\text{Minimum Cost} = \frac{55583}{10 \times 75.633\ldots} + \frac{583 \times 75.633\ldots}{600}$$

$$= \$146.980\ldots$$

$$= \$146.99 \text{ (nearest cent.)}$$

END OF EXAMINATION.